Exponential rates of change – Teacher notes

Activity description

This activity introduces the differentiation of exponential functions.

The skills used are:

- finding gradients by drawing tangents
- finding gradients in Excel by using small incremental changes.

Suitability and Time

Level 3 (Advanced)
1–3 hours

Resources and equipment

Student information sheet, worksheet, rulers, calculators, Excel spreadsheets, slideshow (optional)

Key mathematical language

Exponential growth and decay, gradient, tangent.

Notes on the activity

Part A: before doing this students need to know how to find the gradient of a curve by drawing a tangent.

It is recommended that you divide students into groups, each finding gradients for just one of the graphs and then pooling the results.

Part B: this requires access to Excel; students need to know how to enter spreadsheet formulae and use 'fill-down'.

The information sheet gives spreadsheet formulae which can be used to estimate gradients at points on the curves $y = e^x$ and $y = e^{2x}$ using small increments.

Students are asked to investigate the gradient functions of these and other exponential functions, using the numerical values given by the spreadsheet and graphs drawn from them.

Points for discussion

Students should be encouraged to think about the types of data which lead to graphs illustrating exponential growth and decay.

After students have worked on part A, invite class discussion to work out the connection between the values of y and the value of the gradient at each point.

After students have worked on part B, invite class discussion to work out the connection between columns B and C on the spreadsheets.

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Extensions

Ask students to investigate the gradient functions of functions of the form

$y = ae^{kx}$

$y = e^{kx + c}$

$y = ae^{kx + c}$

where a, k and c are constants.

Answers

x	–2	–1	0	1	2
$y = e^x$	0.1	0.4	1.0	2.7	7.4
Gradient	0.1	0.4	1.0	2.7	7.4

Gradient = e^x

x	–2	–1	0	1	2
$y = e^{2x}$	0.0	0.1	1.0	7.4	54.6
Gradient	0.0	0.3	2.0	14.8	109.2

Gradient = $2e^{2x}$

x	–2	–1	0	1	2
$y = e^{-x}$	7.4	2.7	1.0	0.4	0.1
Gradient	–7.4	–2.7	–1.0	–0.4	–0.1

Gradient = $-e^{-x}$

x	–2	–1	0	1	2
$y = e^{0.5x}$	0.4	0.6	1.0	1.6	2.7
Gradient	0.2	0.3	0.5	0.8	1.4

Gradient = $0.5e^{0.5x}$

The slideshow includes some of the graphs students are asked to draw.
These could aid class discussion.

Here are some results.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^x\right) = e^x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{2x}\right) = 2e^x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{0.5x}\right) = 0.5e^{0.5x} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x}\right) = -e^{-x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{kx}\right) = ke^{kx} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(ae^{kx}\right) = kae^{kx} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{kx+c}\right) = ke^{kx+c} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(ae^{kx+c}\right) = kae^{kx+c}$$

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